1 Problem

The following was observed for a large insured.

Year	Number of Claims	Average Claim Size
1	100	$5,000$
2	150	$10,000$
3	170	$12,750$

It is assumed that inflation is 8% per year. Also assume that claim size follows a Weibull distribution with parameters $\tau = 2$ and unknown θ. What is $VaR_{99\%}$ for the claim size distribution in year 4 where θ is estimated using the method of moments?

(The following may prove helpful: $\Gamma(0.5) = 1.77, \Gamma(1.5) = 0.89, \Gamma(2.5) = 2.33$)

Answers:
A) $12,600$
B) $17,400$
C) $23,000$
D) $27,100$
E) $32,000$

2 Method

1. Calculate the grand total claims over 3 years.
2. Calculate the average claim size.
3. Calculate the method of moments estimate $\hat{\theta}$.
4. Calculate $VaR_{99\%}$.

3 Grand Total Calculation

We need to compute the grand total of claim size over the 3 years, adjusting for inflation.

Year	Total (1)	Inflation Factor (2)	(1)×(2)
1	$100(5,000) = 500,000$	1.08^3	$629,856$
2	$150(10,000) = 1,500,000$	1.08^2	$1,749,600$
3	$170(12,750) = 2,167,500$	1.08	$2,340,900$

The last column sums to $4,720,356$.

4 Average Claim Size Calculation

$$\frac{4,720,356}{420} = 11,238.94$$

5 Method of Moments Calculation

From equation sheet:

$$E(X) = \theta\Gamma(1 + 1/\tau)$$

Thus,

$$\begin{aligned} 11,238.94 &= \theta(0.89) \\ \hat{\theta} &= 12,628.02 \end{aligned}$$

6 Value-at-Risk Calculation

From equation sheet:

$$VaR_p(X) = \theta[-\ln(1-p)]^{1/\tau}$$

Plugging in $\hat{\theta}$, we get

$$\begin{aligned} VaR_{99\%}(X) &= 12,628.02[-\ln(1-0.99)]^{1/2} \\ &= 27,099.30 \end{aligned}$$

Correct answer: D