1 Problem

Given the following values from a uniform distribution, simulate a value from a binomial($m = 100, q = 0.05$) distribution using a stochastic process.

$$0.44, 0.57, 0.92, 0.65, 0.12$$

Answers:
A) 1
B) 2
C) 3
D) 4
E) 5

2 Method

1. Simulate a series of event intervals s_k from an exponential distribution with mean $1/\lambda_k$.
2. Stop when the total time $t_T = \sum_{k=0}^{T} s_k > 1$. The simulated value is then T.

2.1 Inverse CDF of Exponential

$$\begin{aligned}
F_S(s) &= 1 - e^{-s/\theta} \\
&= 1 - e^{-\lambda s} \\
1 - F_S(s) &= e^{-\lambda s} \\
\ln(1 - F_S(s)) &= -\lambda s \\
s &= -\ln(1-u)/\lambda
\end{aligned}$$

Take away:

$$s_k = -\ln(1-u_k)/\lambda_k$$

2.2 Calculation of λ_k

For the binomial(m, q) distribution:

$$d = \ln(1-q)$$

$$c = -md$$

$$\lambda_k = c + dk$$

3 Answer

3.1 Get λ_k

Figure out our parameters:

$$d = \ln(1 - 0.05) = -0.051293$$

$$c = -100(-0.051293) = 5.1293$$

$$\lambda_k = 5.1293 - 0.051293k$$

3.2 Simulate

Simulate:

$$s_0 = -\ln(1 - 0.44)/(5.1293) = 0.1130$$
$$s_1 = -\ln(1 - 0.57)/(5.1293 - 0.051293) = 0.1662$$
$$s_2 = -\ln(1 - 0.92)/(5.1293 - 0.051293(2)) = 0.5025$$
$$s_3 = -\ln(1 - 0.65)/(5.1293 - 0.051293(3)) = 0.2110$$
$$s_4 = -\ln(1 - 0.12)/(5.1293 - 0.051293(4)) = 0.0260$$

3.3 Check Criterion

$$s_0 + s_1 + s_2 + s_3 = 0.9927$$

$$s_0 + s_1 + s_2 + s_3 + s_4 = 1.0187$$

Since $t_4 = \sum_{k=1}^{4} s_k > 1$ but $t_3 = \sum_{k=1}^{3} s_k < 1$, we simulate a value of 4.

Correct answer: D