Exam 1 Solutions

May 20, 2014

1. **Solution: D**
   First, we compute the requirement for full credibility.

$$N = \left(\frac{y}{k}\right)^2 \left(\frac{\sigma_S}{\mu_S}\right)^2 = \left(\frac{1.645}{0.05}\right)^2 \left(\frac{50,000,000}{20,000^2}\right) = 135.301$$

   We require 135.301 claims in order to achieve full credibility, but we only have data for 100. Therefore, we must use the partial credibility factor

$$Z = \sqrt{\frac{100}{135.301}} = 0.860$$

   Now, our credibility-weighted estimate is

$$0.86\frac{2,250,000}{100} + (1 - 0.86)(20,000) = 22,150$$

2. **Solution: B**
   We see that the PDF for X is given by

$$f(x) = \frac{\Gamma(b+1)}{\Gamma(1)\Gamma(b)}(1-x)^{b-1}$$

   We also know that

$$E(X) = \frac{1}{b+1}$$

   Thus, bias is equal to

$$\frac{1}{b+1} - \frac{1}{b} = -\frac{1}{b(b+1)}$$

We can similarly compute the variance of X.

$$Var(X) = E(X^2) - \left(\frac{1}{b+1}\right)^2 = \frac{1(2)}{(b+1)(b+2)} - \left(\frac{1}{b+1}\right)^2$$

The mean-square error is given by

$$MSE(X) = Bias^2(X) + Var(X)$$

This is equal to

$$\left(\frac{1}{b(b+1)}\right)^2 + \frac{2}{(b+1)(b+2)} - \left(\frac{1}{b+1}\right)^2$$

3. **Solution: E**
The mean of X is

$$E(X) = \frac{3\theta}{3-1} = \frac{20+35+70+90}{4} = 53.75$$

Solving for θ, we get $\hat{\theta} = 35.83$. Next,

$$E[(X-30)_+] = E(X) - E[(X \wedge 30)] = 53.75 - [53.75 - \frac{35.83^3}{2(30)^2}] = 25.55$$

4. **Solution: B**
The kernel distribution function is specified by:

$$K_y(x) = \begin{cases} 0 & x < y-7 \\ \dfrac{x-y+7}{2(7)} & y-7 \leq x \leq y+7 \\ 1 & x > y+7 \end{cases}$$

Note that $K_{80}(95) = 1$. Furthermore, $K_{90}(95) = \frac{95-90+7}{2(7)} = 0.8571$, $K_{95}(95) = \frac{95-95+7}{2(7)} = 0.5$, and $K_{100}(95) = \frac{95-100+7}{2(7)} = 0.1429$. Combining these, we get our estimate from a weighted average:

$$\frac{1}{5}(1) + \frac{1}{5}(0.8571) + \frac{1}{5}(0.5) + \frac{2}{5}(0.1429) = 0.5286$$

5. **Solution: C**
The bias is given by

$$E(kX) - \theta = 3k\theta - \theta = (3k-1)\theta$$

Since we know that

$$MSE(\theta) = bias^2 + Var(\hat{\theta})$$

we deduce $Var(\hat{\theta}) = 3bias^2 - k^2$ or $Var(\hat{\theta}) + k^2 = 3bias^2$. Thus,

$$Var(\hat{\theta}) + k^2 = k^2 Var(X) + k^2 = k^2(5\theta^2 - 1) + k^2 = 5k^2\theta^2$$

Thus, we get

$$5k^2\theta^2 = 3(3k-1)^2\theta^2 \Longrightarrow -22k^2 + 18k - 3 = 0$$

We find that $k = 0.233$ or $k = 0.585$.

6. **Solution: D**
Since there are 2 parameters to estimate, we start by calculating the empirical first and second empirical moments.

$$E(X) = \sum xp(x) = \frac{7000}{20,000}(0) + \frac{7000}{20,000}(1) + \frac{4000}{20,000}(2) + \frac{2000}{20,000}(3) = 1.05$$

$$E(X^2) = \sum x^2 p(x) = \frac{7000}{20,000}(0^2) + \frac{7000}{20,000}(1^2) + \frac{4000}{20,000}(2^2) + \frac{2000}{20,000}(3^2) = 2.05$$

The theoretical moments are as follows:

$$E(X) = mp$$

$$E(X^2) = Var(X) + E(X)^2 = mp(1-p) + (mp)^2$$

Substituting the first equation in the second, we get

$$2.05 = mp(1-p) + (mp)^2 = 1.05(1-p) + 1.05^2$$

Solving for p, we get $p = 0.0976$. Then, substituting back into the first moment, we have $1.05 = m(0.0976)$ which implies $m = 10.7582$, or $m = 11$.

Then, we have the log-likelihood to be:

$$
\begin{aligned}
l &= 7000 \log p(0) + 7000 \log p(1) + 4000 \log p(2) + 2000 \log p(3) \\
&= 7000 \log \left(\binom{11}{0} (1 - 0.0976)^{11} \right) + 7000 \log \left(\binom{11}{1} (0.0976)(1 - 0.0976)^{10} \right) \\
&\quad + 4000 \log \left(\binom{11}{2} (0.0976)^2 (1 - 0.0976)^9 \right) \\
&\quad + 2000 \log \left(\binom{11}{3} (0.0976)^3 (1 - 0.0976)^8 \right) \\
&= -26274.7
\end{aligned}
$$

7. **Solution: B**

We appeal to the Central Limit Theorem to conclude that $\bar{X}$ is approximately Normal with mean μ and variance $\sigma^2/100$. For a 95% confidence interval, we need 2.5% in each tail, so our critical value (from the Exam Tables) is 1.96.

The confidence interval is then

$$\bar{X} \pm 1.96 \left(\frac{\sigma}{\sqrt{100}} \right) = 100 \pm 1.96 \left(\frac{\sqrt{625}}{\sqrt{100}} \right) = (95.1, 104.9)$$

8. **Solution: D**

First, $\hat{\mu} = 4.5$ from the sample of two values.

Bootstrapping would yield the following combination of results (disregarding order):

(a) $\{2, 7\}$ with probability 1/2

(b) $\{2, 2\}$ with probability 1/4

(c) $\{7, 7\}$ with probability 1/4

Treating each of these as samples, in the first case, $\bar{X} = 4.5$, in the second case, $\bar{X} = 2$, and in the third case, $\bar{X} = 7$. Thus, we compute MSE as follows:

$$\widehat{MSE}(\bar{X}) = \frac{1}{4}(2 - 4.5)^2 + \frac{1}{2}(4.5 - 4.5)^2 + \frac{1}{4}(7 - 4.5)^2 = 3.125$$

9. **Solution: A**
Recall that $\hat{\lambda}_{MLE} = (\sum n_j)/(\sum e_j)$. Summing across all years, we get $\hat{\lambda}_{MLE} = 0.349$.

The number of losses per year has the same mean and variance of $e_j\hat{\lambda}$.

Thus,

Year	Observed (n_j)	Expected (E_j)	$(O - E_j)^2/E_j$
1	300	349	6.88
2	500	698	56.17
3	700	523	59.90
4	700	628	8.25

Summing the right hand column, we get a test statistic of 131.2. Here, we have the degrees of freedom equal to $4 - 1 = 3$ (since one parameter λ was estimated). The critical value for a 0.001 significance level is 16.2.

10. **Solution: C**
The number of employees that miss work in a day follows a binomial distribution with parameters $m = 100$ and $q = 0.05$.

Using this distribution, we find that

$$Pr(X = 0) = \binom{100}{0}(1 - 0.05)^{100} = 0.00592$$

$$Pr(X = 1) = \binom{100}{1}0.05^1(1 - 0.05)^{99} = 0.03116$$

Similarly, $Pr(X = 2) = 0.08118$ and $Pr(X = 3) = 0.13958$.

From these numbers, we deduce that if our uniform value $u < 0.00592$, we simulate a 0. If $0.00592 \leq u < 0.00592 + 0.03116 = 0.03708$, we simulate a 1. If $0.03708 \leq u < 0.03708 + 0.08118 = 0.11826$, we simulate a 2, and if $0.11826 \leq u < 0.11826 + 0.13958 = 0.25784$, we simulate a 3.

Thus, our simulated values are:

$$1, 2, 3$$

The mean of these values is 2.

11. **Solution: E**
Based on $P_0(x)$, we compute the expected values in each interval by multiplying $n = 110$ (the total number of observations) by $P_0(x)$. Thus,

# Claims	Observed count (O_j)	Expected count (E_j)	$\frac{(O_j - E_j)^2}{E_j}$
$0 \leq x < 2$	55	$(0.5)(110) = 55$	$\frac{(55-55)^2}{55} = 0$
$2 \leq x < 4$	30	33	0.2727
$4 \leq x < 6$	25	22	0.4091

Summing the last column, we get the test statistic of 0.6818. Our test statistic has $df = 3 - 1 = 2$. This gives us a p-value of greater than 0.1.

12. **Solution: D**
Let R denote the event that it rains this afternoon, and let U denote the event that Luis brought his umbrella. Then we compute using Bayes rule:

$$Pr(U|R) = \frac{Pr(U,R)}{Pr(R)} = \frac{Pr(R|U)Pr(U)}{Pr(R|U)Pr(U) + Pr(R|notU)Pr(notU)}$$

This evaluates to

$$\frac{0.9(0.1)}{0.9(0.1) + (0.02)(1 - 0.1)} = 0.8333$$

13. **Solution: A**
The posterior distribution is given by

$$\begin{aligned} f(\lambda|n) &\propto f(n|\lambda)\pi(\lambda) \\ &= \frac{e^{-\lambda}\lambda^n}{n!} \cdot \frac{1}{10} e^{-\lambda/10} \\ &\propto e^{-\lambda(1+1/10)}\lambda^n \end{aligned}$$

Note that this is in the form of a Gamma distribution with $\alpha = n + 1$ and $\theta = 10/11$. The mean of this distribution is given by the Exam Tables: $\alpha\theta = (n + 1)(10/11) = 6(10/11) = 5.454$.

14. **Solution: A**
We know that

$$E[(X-1000)_+] = E(X) - E[(X \wedge 1000)]$$

From the formula sheet, we see that

$$E(X) = \frac{\alpha\theta}{\alpha - 1}$$

and

$$E[(X \wedge 1000)] = \frac{\alpha\theta}{\alpha-1} - \frac{\theta^\alpha}{(\alpha-1)x^{\alpha-1}}$$

Thus, subtracting the two, we get:

$$E[(X-1000)_+] = \frac{\theta^\alpha}{(\alpha-1)x^{\alpha-1}} = \frac{500^2}{1000} = 250$$

15. **Solution: C**
We would prefer a model with a higher chi-square goodness-of-fit p-value, or a lower goodness-of-fit test statistic. All other statements are true.

16. **Solution: C**
The likelihood distribution for the monthly rate parameter λ is given by

$$L(\lambda) = \prod_{i=1}^{2} \frac{e^{-\lambda}\lambda^{x_i}}{x_i!}$$

Taking logs, we get

$$l(\lambda) = -2\lambda + \sum x_i \log\lambda - \log(\prod x_i!)$$

This is maximized when

$$\lambda = \bar{X} = 1$$

$Y = X_1 + X_2$ follows a Poisson distribution with rate parameter $\lambda_1 + \lambda_2 = 2\lambda = 2(1) = 2$. Thus, we compute

$$Pr(Y > 1) = 1 - Pr(Y=0) - Pr(Y=1) = 1 - e^{-2} - \frac{e^{-2}2^1}{1!} = 0.594$$

17. **Solution: B**
Recall that observed information $\hat{I}(\theta) = -\frac{d^2 l(\theta|x)}{d\theta}\Big|_{\theta=\hat{\theta}}$. Thus, we find the second derivative of the log-likelihood.

$$L(\lambda) = \prod_{i=1}^{2} \frac{e^{-\lambda}\lambda^{x_i}}{x_i!}$$

Taking logs, we get

$$l(\lambda) = -2\lambda + \sum x_i \log \lambda - \log(\prod x_i!)$$

$$l'(\lambda) = -2 + 2 \cdot \frac{\bar{X}}{\lambda}$$

From here, we see that the maximum likelihood estimate $\hat{\lambda} = \bar{X} = 1$.

$$l''(\lambda) = -2 \cdot \frac{\bar{X}}{\lambda^2}$$

We now plug in our values for $\bar{X}$ and $\hat{\lambda}$ to get

$$\hat{I}(\lambda) = 2 \cdot \frac{1}{1} = 2$$

18. **Solution: C**
A correct restatement of C is

$$Z = \frac{N^2\kappa^2}{N^2\kappa^2 + N\gamma^2}$$

19. **Solution: D**
Recall that

$$r_j = \#\{x_i \geq y_j\} + \#\{u_i \geq y_j\} - \#\{d_i \geq y_j\}$$

At $y_1 = 1.2$, the risk set $r_1 = 3 + 2 - 1 = 4$, and $s_1 = 1$. At $y_2 = 1.3$, the risk set $r_2 = 2 + 2 - 1 = 3$, and $s_2 = 1$.

The Kaplan-Meier estimator is thus

$$\hat{S}(2) = \frac{4-1}{4} \cdot \frac{3-1}{3} = 0.5$$

20. **Solution: E**

The formula for Greenwood's approximation is

$$\widehat{Var}(\hat{S}(2)) = \hat{S}(2)^2 \sum_{i=1}^{j} \frac{s_i}{r_i(r_i - s_i)}$$

From the previous problem, we found that $\hat{S}(2) = 0.5$, $r_1 = 4$, $r_2 = 3$, and $s_1 = s_2 = 1$.

Greenwood's approximation is thus

$$\widehat{Var}(\hat{S}(2)) = 0.5^2 \left(\frac{1}{4(4-1)} + \frac{1}{3(3-1)} \right) = 0.0625$$

21. **Solution: E**

Since we have complete observations for X, our likelihood function will be comprised of a product of densities given by $f(1)f(2)f(2)$. We therefore start by computing the density $f(x)$, using the relationship $f(x) = -S'(x)$.

$$f(x) = -S'(x) = 2\theta^2(\theta + 3x)^{-3}(3) = \frac{6\theta^2}{(\theta + 3x)^3}$$

Now our likelihood becomes

$$L(\theta) = \frac{6\theta^2}{(\theta+3)^3} \left(\frac{6\theta^2}{(\theta+6)^3} \right)^2$$

Taking logs, we get

$$l(\theta) = \log(6^3\theta^6) - 3\log(\theta + 3) - 6\log(\theta + 6)$$

Taking the derivative, we then get

$$l'(\theta) = \frac{6}{\theta} - \frac{3}{\theta+3} - \frac{6}{\theta+6} = 0$$

This factors out to

$$\frac{3(\theta^2 - 6\theta - 36)}{\theta(\theta+3)(\theta+6)} = 0$$

With the constraint that $\theta > 0$, we find that

$$\theta^2 - 6\theta - 36 = 0$$

The only positive solution (via the quadratic formula), is then $\theta = 3 + 3\sqrt{5} = 9.71$

22. **Solution: C**
We have

$$\mu_f = E(N) = 5(0.1) = 0.5$$
$$\sigma_f^2 = Var(N) = 5(0.1)(0.9) = 0.45$$

Then,

$$\sigma_{PP}^2 = \mu_f \sigma_S^2 + \mu_S^2 \sigma_f^2 = 0.5(1050) + 200^2(0.45) = 18,525$$

23. **Solution: C**

$$EPV = E(Var(N|\lambda)) = E(\lambda) = 0.7p + 1.3(1-p) = 1.3 - 0.6p$$

$$\begin{aligned} VHM &= Var(E(N|\lambda)) \\ &= Var(\lambda) \\ &= E(\lambda^2) - [E(\lambda)]^2 \\ &= 0.7^2 p + 1.3^2(1-p) - (1.3 - 0.6p)^2 \\ &= -0.36p^2 + 0.36p \end{aligned}$$

We then have

$$k = \frac{EPV}{VHM} = \frac{1.3 - 0.6p}{-0.36p^2 + 0.36p}$$

Next, since we are interested in modeling frequency, $N = 1$ is the number of exposures and

$$z = \frac{1}{1+k} = \frac{1}{1 + \frac{1.3-0.6p}{-0.36p^2+0.36p}}$$

24. **Solution: B**
Let C denote the event that a patient has cancer, and let B be a random variable for the number of barks.

We have

$$Pr(C) = \sum_{b=0}^{3} Pr(C|B = b)Pr(B = b)$$

This calculation yields $0.01(0.5)+(0.3)(0.2)+0.7(0.2)+0.9(0.1) = 0.295$

25. **Solution: A**
Let S denote aggregate losses and let N denote claim count.

$$E(S|\lambda, \theta) = 3\lambda\theta$$

$$\begin{aligned} Var(S|\lambda,\theta) &= Var[E(S|N)|\lambda,\theta] + E[Var(S|N)|\lambda,\theta] \\ &= Var(3N\theta|\lambda,\theta) + E[3N\theta^2|\lambda,\theta] \\ &= 3^2\lambda(\theta^2) + 3\theta^2\lambda \\ &= 12\lambda\theta^2 \end{aligned}$$

Now since λ and θ are independent,

$$EPV = E(12\lambda\theta^2) = 12E(\lambda)E(\theta^2) = 12(2)(5+5^2) = 720$$

$$\begin{aligned} VHM &= Var(3\lambda\theta) \\ &= 9[E(\lambda^2\theta^2) - (E(\lambda)E(\theta))^2] \\ &= 9[E(\lambda^2)E(\theta^2) - (2(5))^2] \\ &= 9((5+5^2)2(2^2) - 100) \\ &= 1260 \end{aligned}$$

Thus, $k = EPV/VHM = 720/1260 = 0.5714$.

26. **Solution: A**
Let $Y = \max X_i$. Then, the PDF of Y is:

$$\begin{aligned} F_Y(y) &= Pr(Y \leq y) \\ &= Pr(X_1 \leq y, X_2 \leq y, \ldots, X_n \leq y) \\ &= F_X(y)^n \\ &= \left(\frac{y}{\theta}\right)^n \end{aligned}$$

Taking derivatives,

$$f_Y(y) = \frac{n}{\theta^n} y^{n-1}$$

The variance of Y is:

$$\begin{aligned}
Var(Y) &= E(Y^2) - [E(Y)]^2 \\
&= \frac{n}{\theta^n} \int_0^\theta y^2 y^{n-1} dy - \left(\frac{n}{\theta^n} \int_0^\theta y y^{n-1} dy \right) \\
&= \frac{n}{n+2}\theta^2 - \left(\frac{n}{n+1} \right)^2 \theta^2 \\
&= \frac{n(n+1)^2 - n^2(n+2)}{(n+2)(n+1)^2}\theta^2 \\
&= \frac{n^3 + 2n^2 + n - n^3 - 2n^2}{(n+2)(n+1)^2}\theta^2 \\
&= \frac{n}{(n+2)(n+1)^2}\theta^2
\end{aligned}$$

Taking the limit as $n \to \infty$, we get that the asymptotic variance is 0.

27. **Solution: D**
The log-likelihood function for the exponential parameter θ is:

$$\begin{aligned}
l(\theta) &= \log \prod \left(\frac{1}{\theta} e^{-x_i/\theta} \right) \\
&= -n \log \theta - \left(\frac{\sum x_i}{\theta} \right) \\
&= -n \log \theta - \left(\frac{n\bar{X}}{\theta} \right)
\end{aligned}$$

We take the derivative with respect to θ to get

$$l'(\theta) = -\frac{n}{\theta} + \frac{n\bar{X}}{\theta^2}$$

Setting this to 0, we solve for θ to find $\hat{\theta} = \bar{X} = 3500$.

28. **Solution: D**

k	$\frac{kn_k}{n_{k-1}}$
0	
1	0.7442
2	0.75
3	2
4	2.5

First, $\frac{kn_k}{n_{k-1}}$ is an increasing function of k, which implies that negative binomial is appropriate. The sample mean is 1, smaller than the sample variance of 1.3 (either using a denominator of n or $n-1$ wouldn't matter here), which also suggests negative binomial would be a good fit.

29. **Solution: B**
The PDF for the claims is

$$f(x) = \frac{0.7}{x}\left(\frac{x}{\theta}\right)^{0.7} \exp\{-\left(\frac{x}{\theta}\right)^{0.7}\}$$

Thus, our joint likelihood becomes

$$L(\theta) = \left[\frac{0.7^4}{\prod_{i=1}^4 x_i}\prod_{i=1}^4\left(\frac{x_i}{\theta}\right)^{0.7}\right]\exp\{-\theta^{-0.7}\sum_{i=1}^5 x_i^{0.7}\}$$

Taking logs, we get

$$l(\theta) = 4\log(0.7) + (0.7-1)\sum_{i=1}^4 \log(x_i) - 0.7(4)\log\theta - \theta^{-0.7}\sum_{i=1}^5 x_i^{0.7}$$

Taking the derivative, we now have

$$l'(\theta) = -\frac{2.8}{\theta} + 0.7\theta^{-1.7}\sum_{i=1}^5 x_i^{0.7}$$

Setting this to 0, we solve for θ to get $\hat{\theta} = 564.83$.

30. **Solution: C**
Let Y denote the number of claims next year.

$$E(Y|p) = 0\left(\frac{p}{2}\right) + 1(p^2) + 2(1 - p/2 - p^2) = 2 - p - p^2$$

The posterior distribution of p can be computed as:

$$f(p|x) = \frac{f(x|p)f(p)}{f(x)} = \frac{p^2(1-p)}{\int_0^1 (1-p)p^2 dp} = 12p^2(1-p)$$

From this, we compute the first and second moments of the posterior distribution:

$$E_{p|x}(p) = \int_0^1 12p^3(1-p)dp = \frac{12}{20}$$

$$E_{p|x}(p^2) = \int_0^1 12p^4(1-p)dp = \frac{12}{30}$$

Recall that

$$E(Y|X) = E_{p|X}(E(Y|p)) = E_{p|X}(2-p-p^2)$$

This becomes

$$2 - \frac{12}{20} - \frac{12}{30} = 1$$

31. **Solution: C**
$F_{10}(6)$ lies between $F_{10}(5)$ and $F_{10}(7)$, which are equal to 0.6 and 0.7 accordingly. Then,

$$F_{10}(6) = \frac{7-6}{7-5}F_{10}(5) + \frac{6-5}{7-5}F_{10}(7) = 0.65$$

32. **Solution: C**
We first find $F_i(3000)$ for $i = 1, 2$, and 3.

$$F_1(3000) = 1 - e^{-(3000/1000)^2} = 0.9999$$

$$F_2(3000) = 1 - e^{-(3000/4000)^2} = 0.4302$$

$$F_2(3000) = 1 - e^{-(3000/5000)^2} = 0.3023$$

We then take a weighted average to get:

$$F(3000) = \frac{\lambda_1(0.9999) + \lambda_2(0.4302) + \lambda_3(0.3023)}{\lambda_1 + \lambda_2 + \lambda_3} = 0.5193$$

33. **Solution: A**

We produce the following table:

x	$F^*(x)$	$F_n(x-)$	$F_n(x)$	$\max\{\lvert F_n(x-) - F^*(x)\rvert, \lvert F_n(x) - F^*(x)\rvert\}$
20	0.0607	0	0.2	0.1393
37	0.1824	0.2	0.4	0.2176
92	0.6791	0.4	0.6	0.2791
102	0.7492	0.6	0.8	0.1492
130	0.8884	0.8	1	0.1116

The maximum value in the right column is 0.2791. Our critical value at $\alpha = 0.05$ is $1.36/\sqrt{n} = 0.608$. Our test statistic of 0.2791 is smaller, so we fail to reject the null hypothesis that the Weibull$(1.9, 86)$ distribution is an appropriate fit.

34. **Solution: A**

Recall that the log transformed interval has the following form:

$$\left(\frac{\hat{H}(t)}{v}, \hat{H}(t)v\right)$$

where

$$v = \exp\left\{\frac{1.96\sqrt{\widehat{Var}(\hat{H}(t))}}{\hat{H}(t)}\right\}$$

Multiplying the two endpoints of our interval yields $\hat{H}(t)^2 = 0.65(0.23) = 0.1495$. Thus,

$$\hat{S}(t) = e^{-0.1495} = 0.8611$$

35. **Solution: E**

We know that

$$Pr(X = 2) = \frac{d^2}{dz^2}\frac{P_X(z)}{2!}\bigg|_{z=0}$$

To make our lives easier, we will write $P_X(z)$ as

$$P_X(z) = 0.8^3(1 - 0.2z)^{-3}$$

Taking the first derivative, we have

$$\frac{d}{dz}P_X(z) = 0.8^3(-3)(1-0.2z)^{-4}(-0.2) = 0.3072(1-0.2z)^{-4}$$

Now, taking the second derivative, we have

$$\frac{d^2}{dz^2}P_X(z) = 0.3072(-4)(1-0.2z)^{-5}(-0.2) = 0.24576(1-0.2z)^{-5}$$

Now, we can compute the probability as follows:

$$Pr(X=2) = \left.\frac{0.24576(1-0.2z)^{-5}}{2!}\right|_{z=0} = \frac{0.24576}{2} = 0.1229$$

36. **Solution: D**
The expected payment per loss is calculated by

$$\int_{50}^{\theta} x\frac{1}{\theta}dx = \left.\frac{x^2}{2\theta}\right|_{50}^{\theta} = \frac{\theta}{2} - \frac{1250}{\theta}$$

The average sample payment is simply the average of the 6 observations, equaling 100.

Thus,

$$100 = \frac{\theta}{2} - \frac{1250}{\theta}$$

Solving this yields $\theta = 211.80$.

37. **Solution: C**
We require

$$n \geq \left(\frac{z_{0.05}}{0.02}\right)^2 \frac{s^2}{\bar{X}^2}$$

where $s^2 = p/n(1-p/n)$ and $\bar{X} = p/n$. This simplifies to

$$n \geq \left(\frac{1.645}{0.02}\right)^2 \frac{p(n-p)/n^2}{p^2/n^2} = \left(\frac{1.645}{0.02}\right)^2 \frac{n-p}{p}$$

We can also write this as

$$\frac{np}{n-p} \geq \left(\frac{1.645}{0.02}\right)^2 = 6765.06$$

Plugging in each of the options, we see that C is the only one that satisfies the inequality.

38. **Solution: B**
Recall that the skewness coefficient is given by

$$\gamma_1 = \frac{E(X-\mu)^3}{\sigma^3}$$

We start by computing $\hat{\mu} = \bar{X}$:

$$\bar{X} = \frac{2.4 + 3.6 + 5.2 + 5.3 + 5.7 + 6.9 + 10.1}{7} = 5.6$$

We then compute the sample variance $\hat{\sigma}^2$:

$$\hat{\sigma}^2 = \frac{(2.4-5.6)^2 + (3.6-5.6)^2 + \cdots + (10.1-5.6)^2}{7} = 5.2057$$

Similarly,

$$\hat{E}(X-\mu)^3 = \frac{1}{7} \cdot \sum_i (x_i - \mu)^3 = 7.4949$$

Based on this information, we now have:

$$\gamma_1 = \frac{7.4949}{5.2057^{3/2}} = 0.6310$$

39. **Solution: E**
Note that our formula for expected cost per payment is:

$$E(Y^P) = \frac{(1+r)[E(X) - E(X \wedge \frac{d}{1+r})]}{S_X(\frac{d}{1+r})}$$

For an exponential distribution (from the Exam Tables),

$$E(X) - E(X \wedge \frac{d}{1+r})] = \theta S_X(\frac{d}{1+r})$$

Thus, we end up with $E(Y^P) = \theta(1+r)$, a formula that does not involve the deductible d.

40. **Solution: A**
First, we recognize that the prior of β is Pareto with $\alpha = 3$ and $\theta = 1$.

Note that

$$EPV = E(Var(N|\beta)) = E(2\beta(1+\beta)) = 2\left[\frac{1}{3-1} + \frac{2!}{(3-1)(3-2)}\right] = 3$$

$$VHM = Var(E(N|\beta)) = Var(2\beta) = 4\left[\frac{2!}{2} - \left(\frac{1}{2}\right)^2\right] = 3$$

Therefore $k = EPV/VHM = 1$, and $z = \frac{1}{1+1} = 1/2$. Our estimate is then:

$$0.5(x) + 0.5E(2\beta) = 0.5x + 0.5(2)(1/2) = 0.5(1+x)$$