

Exam C: Theoretical Knowledge and Exercises

6th Edition

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In this manual, we will provide the theoretical underpinnings required for Exam 4/C. Although we strongly recommend reading all the materials listed on the syllabus, it is not always practical or feasible given the time constraints of the exam taking process. Factors such as work, family, friends – sleeping is important too! – led to the necessity of this study guide. Thus, this guide is intended to be a standalone resource to try to impart the knowledge you need to know for the test and lightly touch upon the things you don't need to know. Realize that these tests are not meant to measure mathematical aptitude. Instead, similar types of problems are tested again and again, and are rooted in the same fundamental concepts. Master the concepts and you will master the exam.

The goal of this manual is to prepare you so that you pass the exam with room to spare, but at the same time not overburden you with facts and theorems you do not need. We will expand on the material where we think more detail is useful and condense other sections that we think are not testable (such as proofs). However, these decisions are always, to some extent, at the mercy of the authors. We do our best to make these selections carefully but any manual will always be biased in this way. As such, we recommend the following strategy: work through this manual once thoroughly but do not agonize over one specific part. Almost everyone will find one part or another difficult. What you should agonize over are the practice questions provided. These problems will have to be done again and again until they become second nature. Don't worry if your answers do not match with our solutions to the exact decimal place. Do focus on matching the procedures. On the exam, carry out all calculations to at least five decimal places. We highly recommend working through all of the old exams available on the SOA and CAS websites.

When you see a problem on an exam, you should know within 15 seconds whether or not this is something you will be able to solve. If you don't know how to do it, skip it rather than wasting your time dwelling on it. The hope is that with this approach you will be able to get all the easy problems and then have enough time to tackle some of the shorter hard problems as well. That should be more than sufficient to pass.

Good luck in your preparations. All comments, suggestions and corrections are welcome! Please feel free to contact us via our website at: <http://www.actempire.com/>

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– Greg & Xiaofei

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